The Stokes-Leibenson problem for Hele-Shaw flows

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Let Ω_0 be a bounded simply connected bi-dimensional domain containing the origin(where the source-sink point islocated)such that its boundary Γ_0 is smooth enough.
We have to find at each time t a domain Ω_t of boundary Γ_t such that the normal velocity of each points $s \in \Gamma_t$ is given by

$$s \cdot \nu = \partial_\nu u, \qquad \text{on} \quad \Gamma_t$$

where u satisfies

$$\Delta u = q\delta, \quad \text{in} \quad \Omega_{t,}$$

$$u = 0, \quad \text{on} \quad \Gamma_t.$$

Here, δ is the Dirac distribution at the origin, q is a real constant:$q > 0$ in the source-case, $q < 0$ in the sink-case.
The Helmholtz-Kirchhoff method leads to write this problem as a Cauchy problem for an integro-differential equation which is posed in some Fixed domain of the complex plane.
Theoretical results can be obtained about the existence, the uniqueness andthe behavior of the solution.
Furthermore by using ideas of this method, a discrete model can be built: the "quasi-contour" model which is close to the Wulff scheme for crystalline flows. Some numerical experiments will be presented.