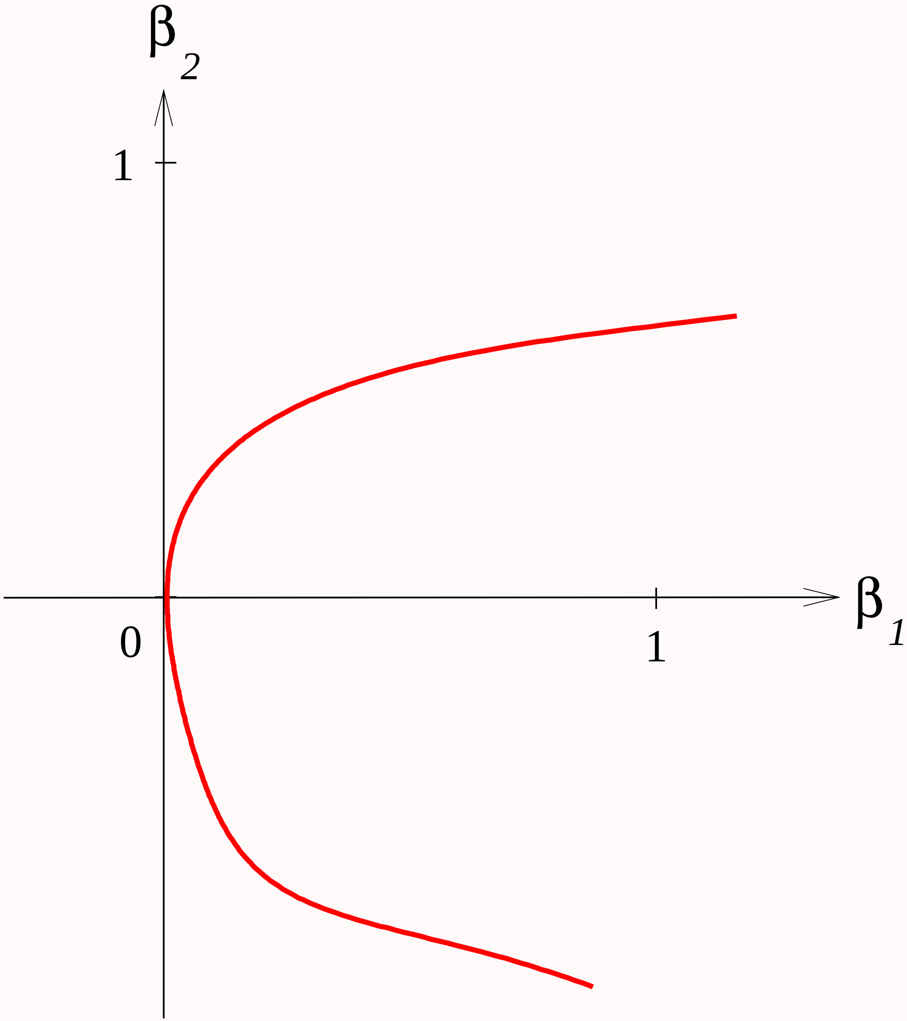
1. Galerkin approximation

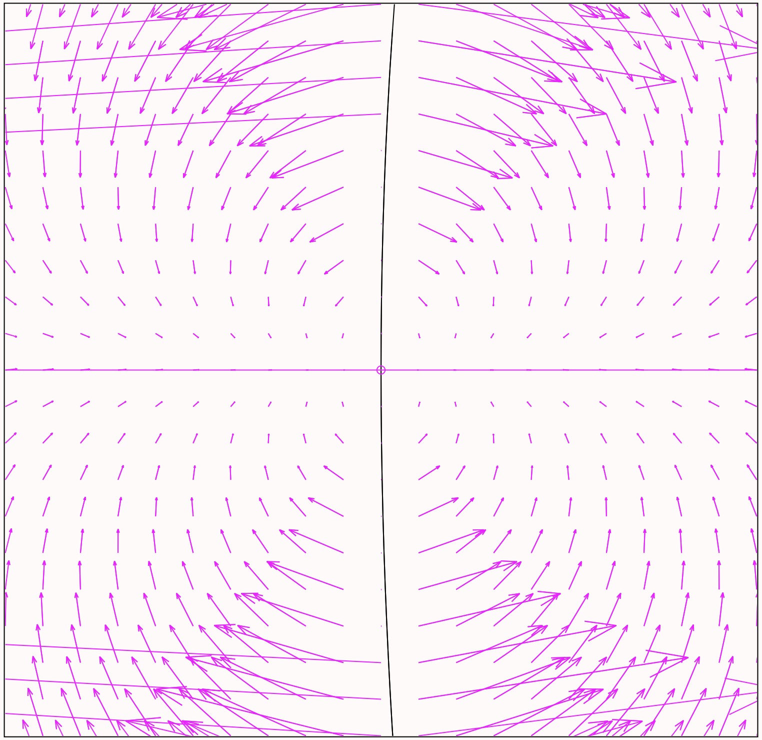
Apply a forward Euler scheme to the system

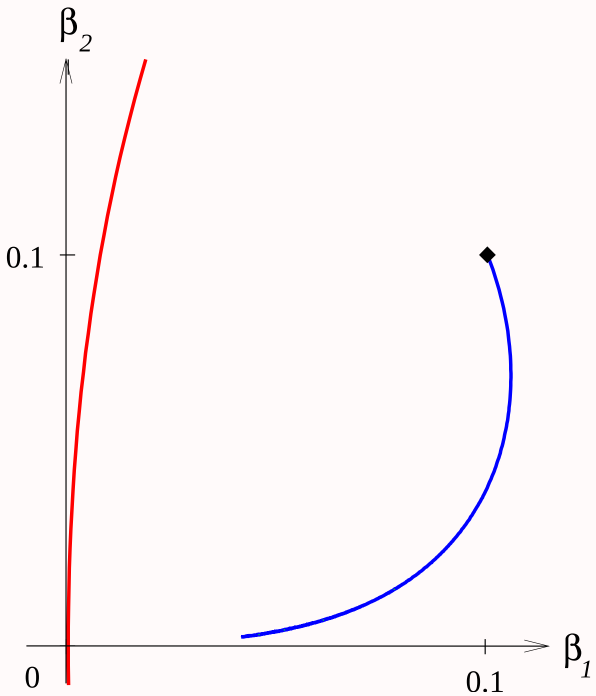
$$2(t+t_0)\langle[(\mathbb{I}-\mathbb{K}(\beta))(\partial_t\beta)]\,(.,t),S_\jmath\rangle = \langle[\mathbb{F}(\beta)](.,t),S_\jmath\rangle, \quad \jmath = 1,2,\ldots,m\,,$$

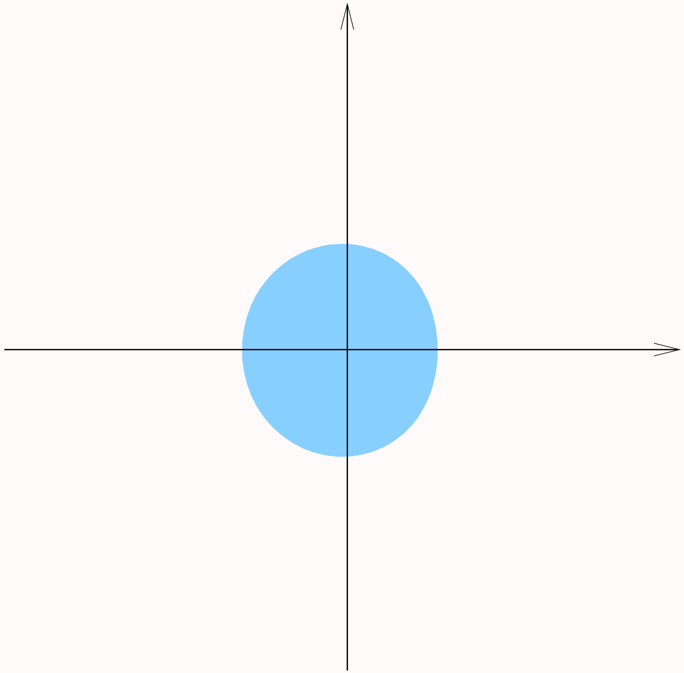
with $\quad \beta(.,t) = \sum_{k=1}^{m} \beta_k(t) S_k.$

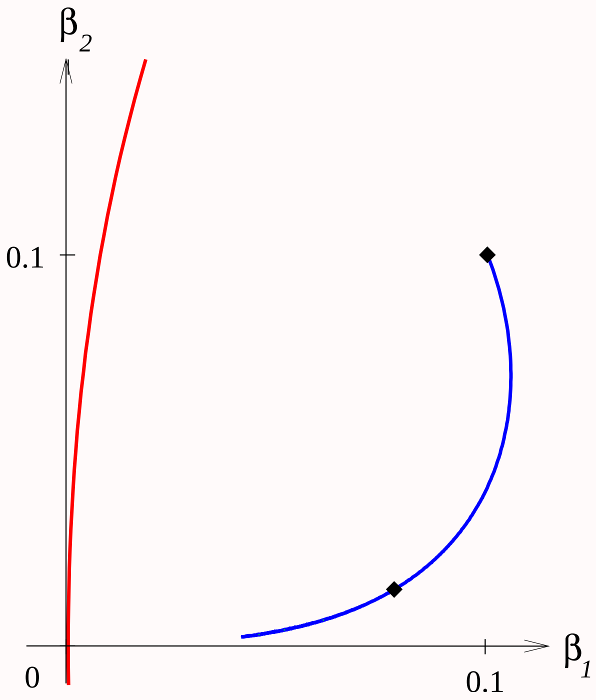
The matrix of this system can be singular...

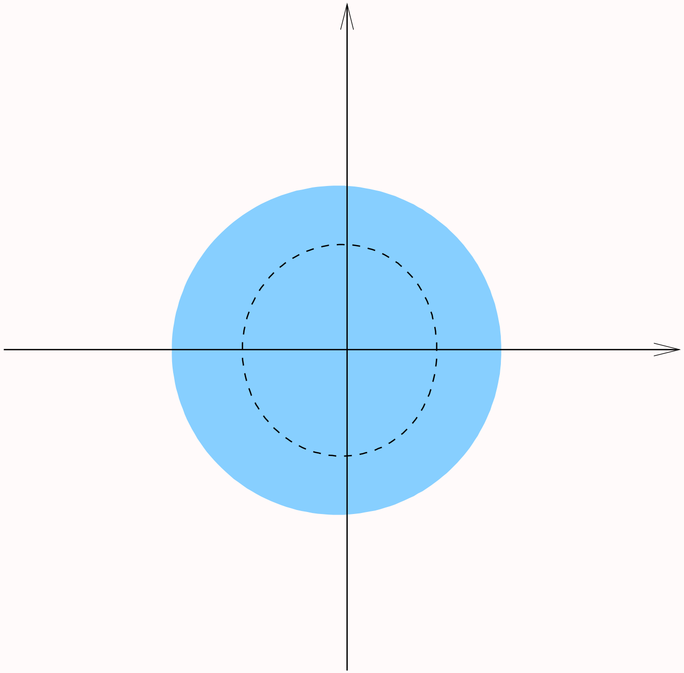

β2
1
0
1
β1

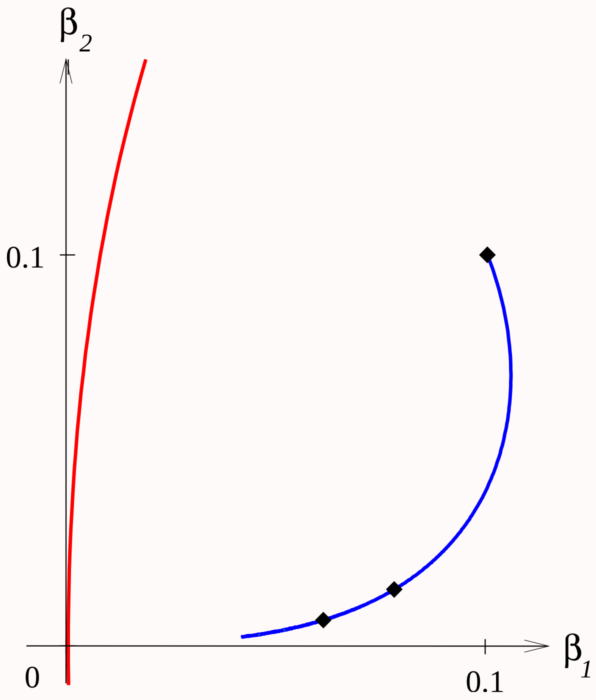


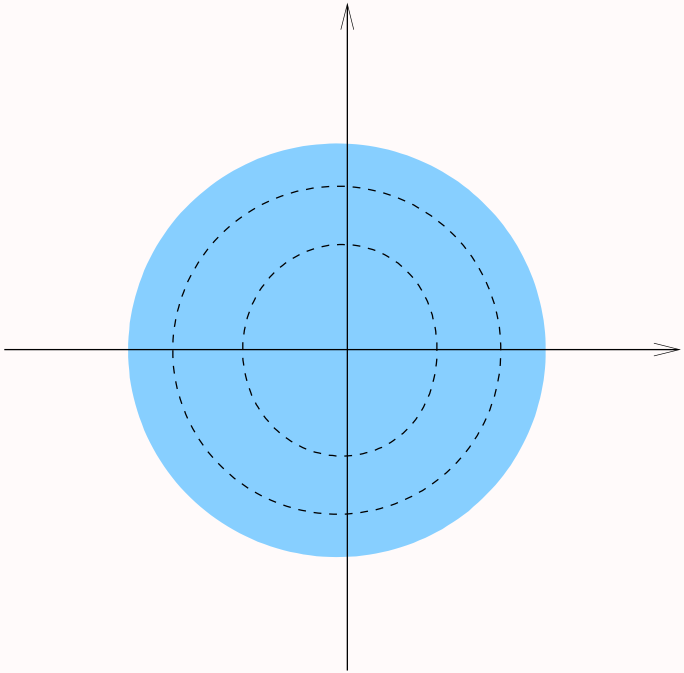

β2
0.1
0
0.1
β1

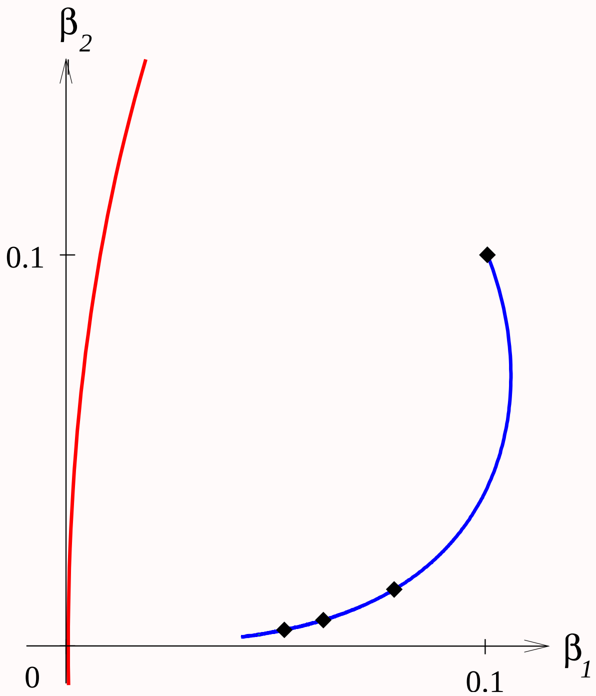


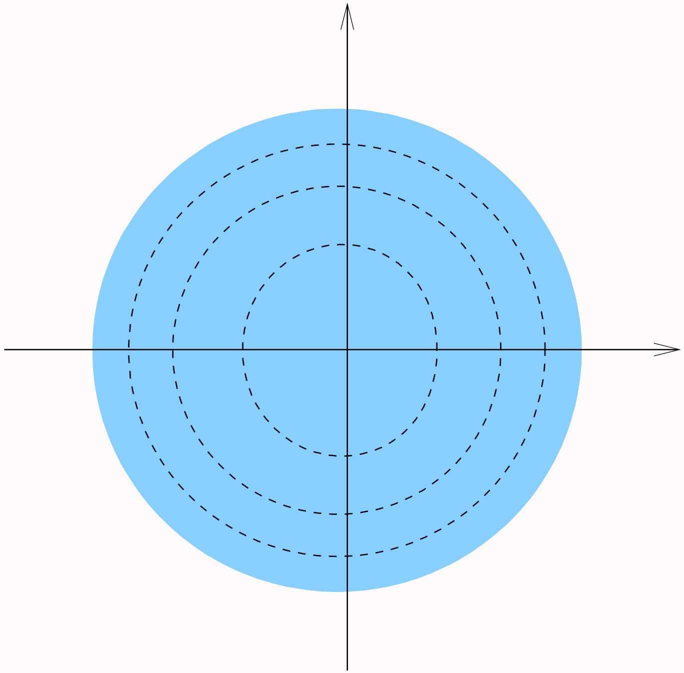

β2
0.1
0
0.1
β1




β2
0.1
0
0.1
β1




β2
0.1
0
0.1
β1



2. Quasi-contour model

The forward Euler scheme can be formally written

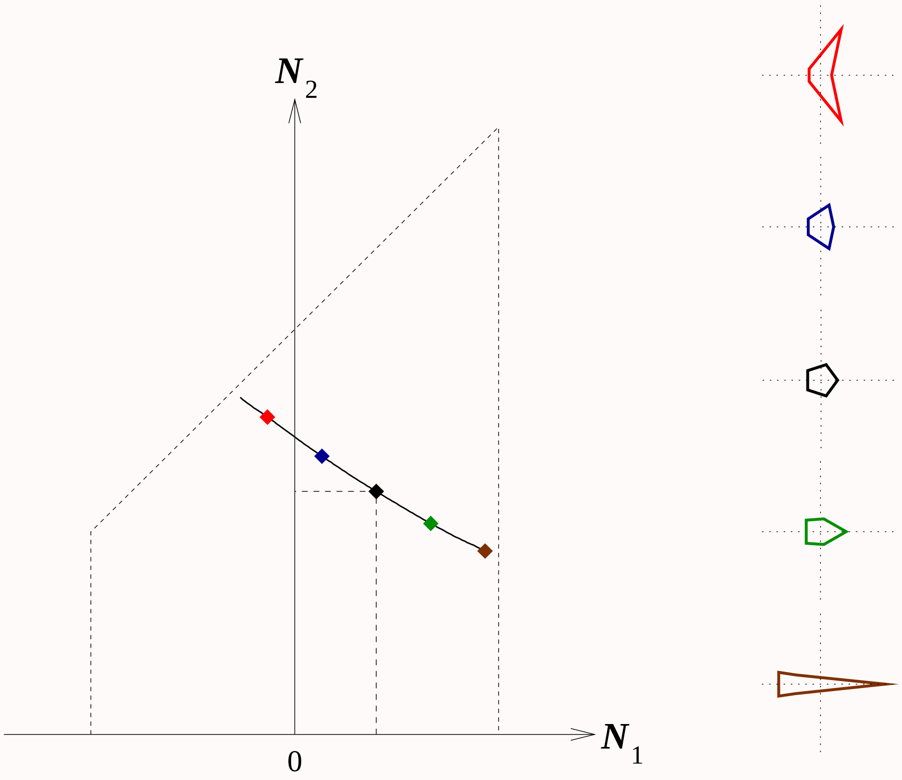
$$\begin{cases} t_{\ell+1} = t_\ell + \delta t_\ell \,, \\ \mathbf{N}^{(\ell+1)} = \mathbf{N}^{(\ell)} + \dfrac{\delta t_\ell}{2t_\ell} Q(\mathbf{N}^{(\ell)})^{-1} P(\mathbf{N}^{(\ell)}) \,, \end{cases} \qquad \ell \in \mathbb{N} \,.$$

- Is $Q(\mathbf{N})$ singular ?
- Behavior of quasi-contours for some values of m and σ.

$m = 2, \sigma = (2/5, 4/5).$

Domain of computation

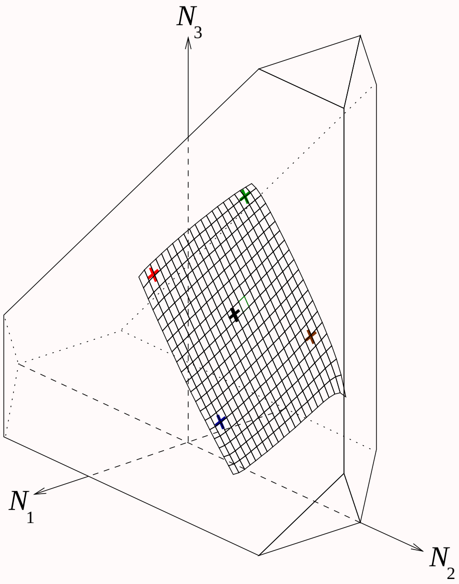
$$\begin{cases} -\dfrac{\pi}{2} < N_1 < \dfrac{\pi}{2}\,, \\ 0 < N_2 < N_1 + \pi\,. \end{cases}$$

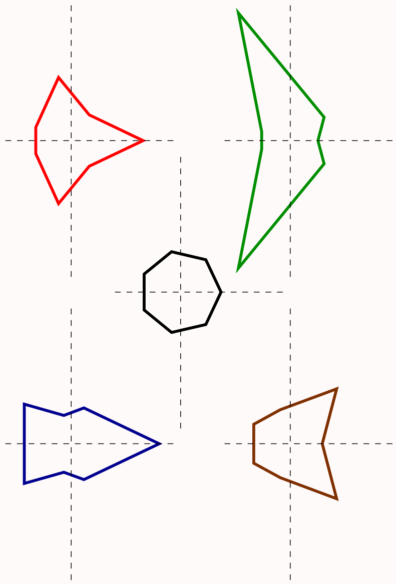


$m = 3, \sigma = (2/7, 4/7, 6/7).$

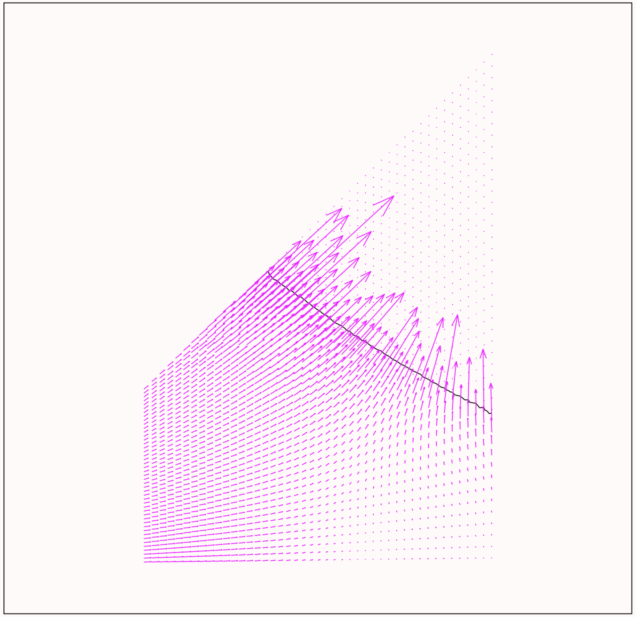
Domain of computation

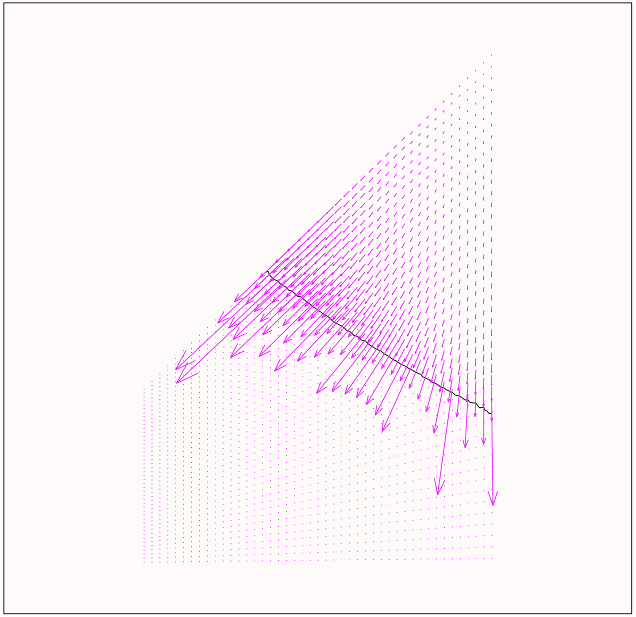
$$\begin{cases} -\frac{\pi}{2} < N_1 < \frac{\pi}{2}\,, \\ N_1 - \pi < N_2 < N_1 + \pi\,, \\ N_3 - \pi < N_2 < N_3 + \pi\,, \\ 0 < N_3 < 2\pi\,. \end{cases}$$



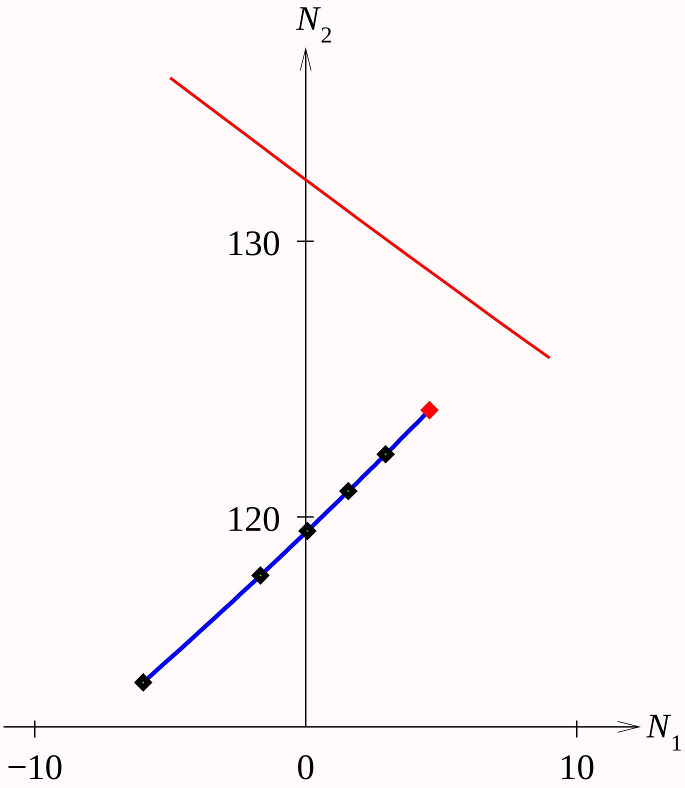


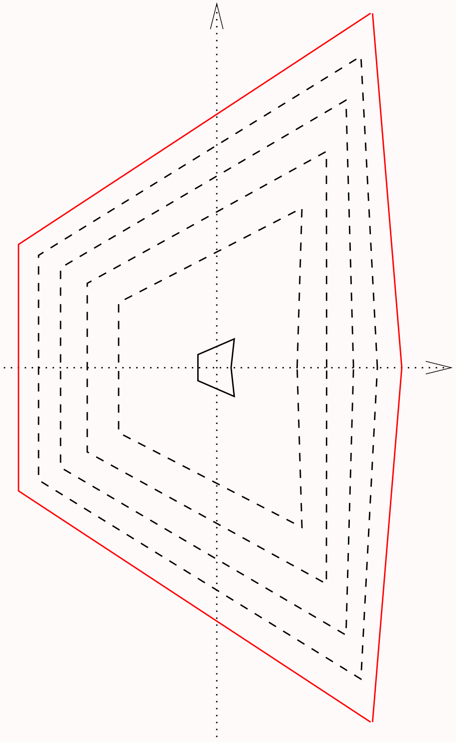
$m = 2, \sigma = (2/5, 4/5).$



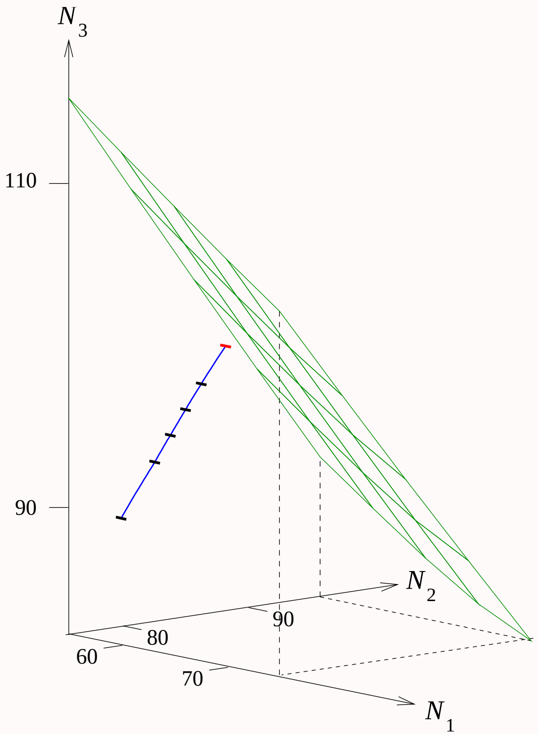


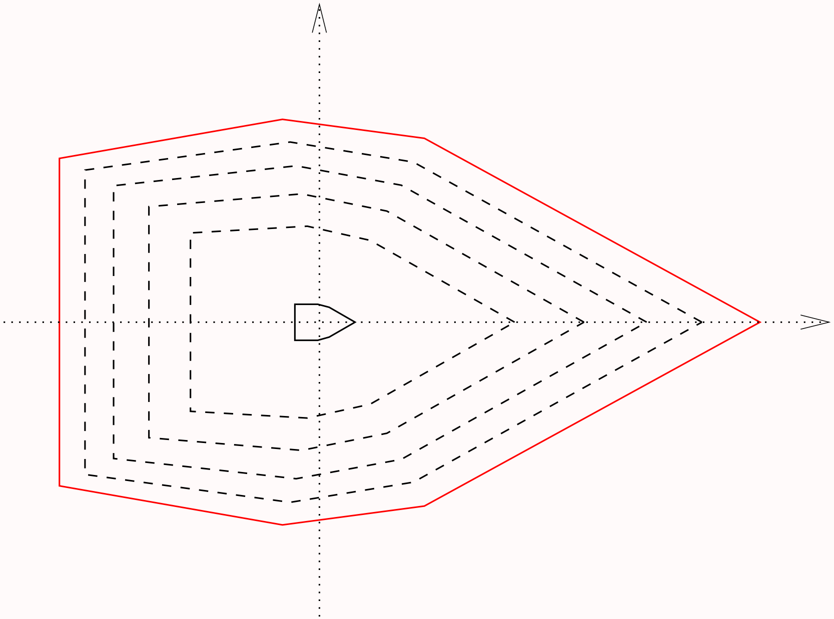
$m = 2, \sigma = (2/5, 4/5).$



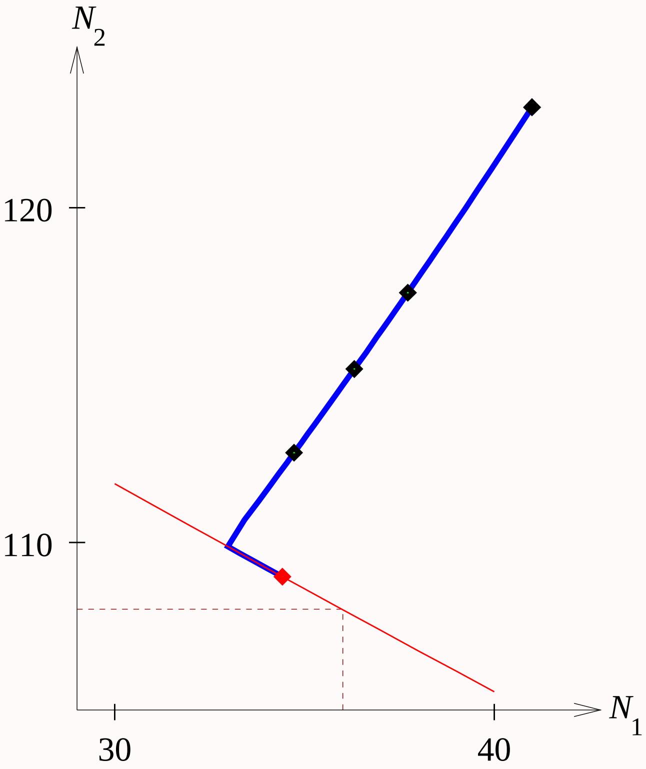


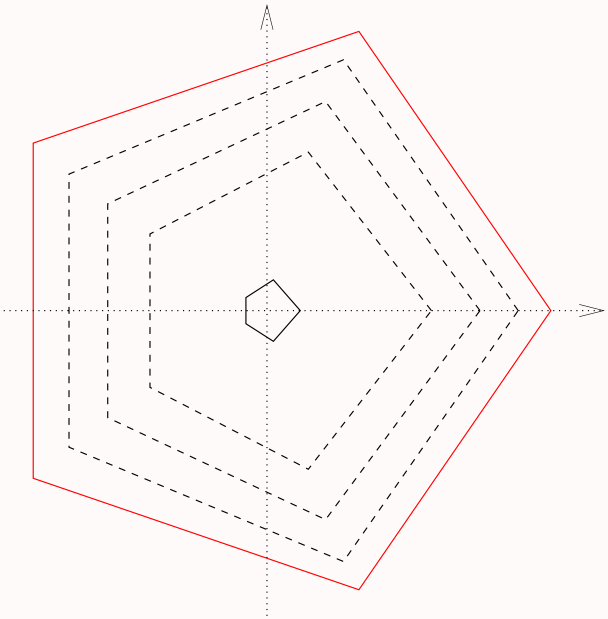
$m = 3, \sigma = (2/7, 4/7, 6/7).$



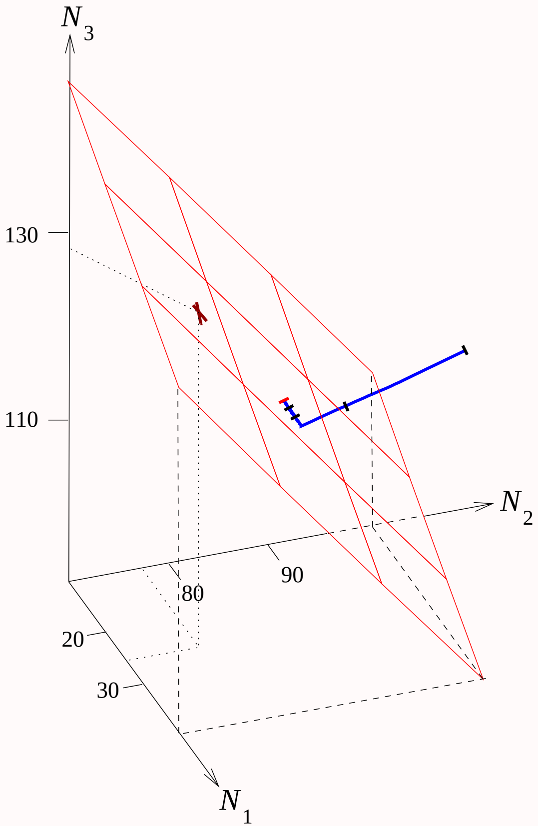


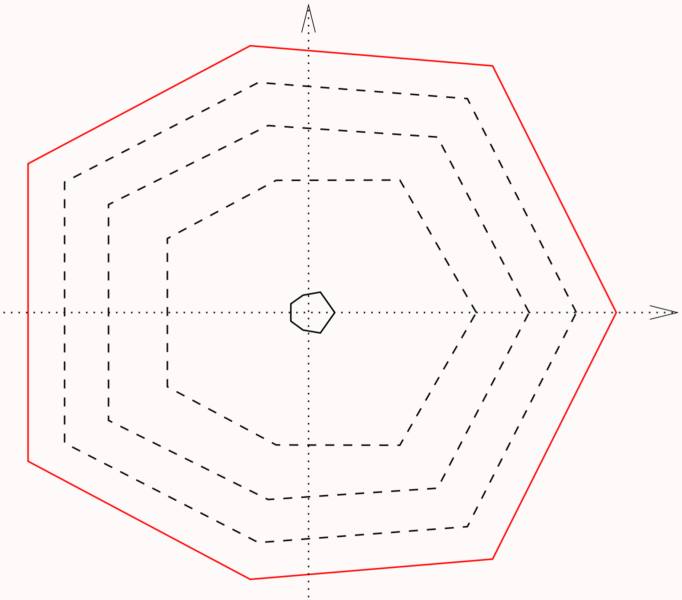
$m = 2, \sigma = (2/5, 4/5).$



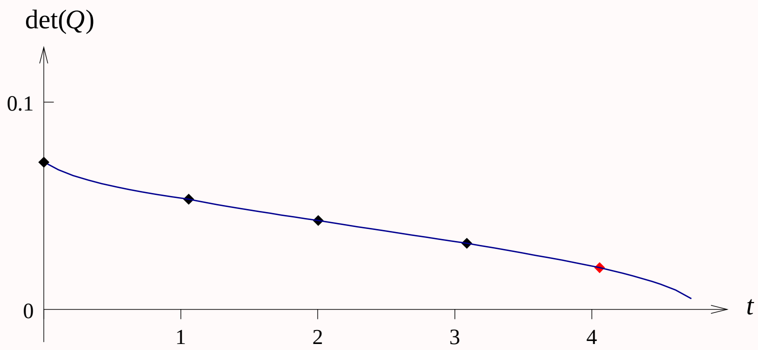


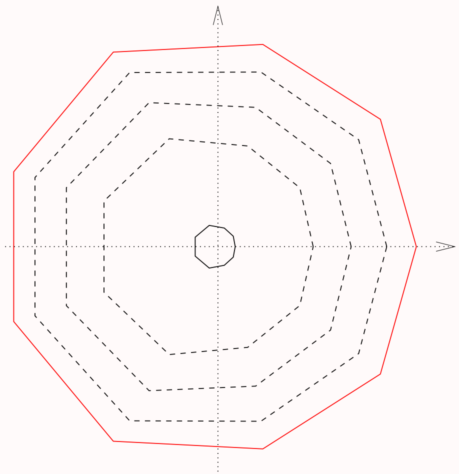
$m = 3, \sigma = (2/7, 4/7, 6/7)$.





$m = 4, \sigma = (2/9, 4/9, 6/9, 8/9).$





$m = 5, \sigma = (2/11, 4/11, 6/11, 7/11, 9/11)$.